Name: ______________________________________ Date: _____________

Midterm Exam Review Algebra II

Know:

- The symbolic difference between an inequality and an equation
- The graphical difference between an inequality and an equation
- The difference between an expression and an equation
- The key pieces of linear programming: variables, inequalities, points of interest (vertices of feasible region), the difference between a restriction (inequality) and the expression that you are trying to maximize or minimize, how to use PoI to minimize or maximize, how to find the App (Inequalz) and utilize it to efficiently handle linear programming questions
- The difference between a function and a non-function in terms of inputs and outputs, tables, and graphs
- Linear Equations: (variables are x and y)
  - Slope Intercept Form: y = mx + b (m is slope, b is y-intercept)
  - Standard Form: Ax + By = C
  - x = "a number" is a vertical line. y = "a number" is a horizontal line.
- Quadratic Equations: (variables are x and y)
  - Intercept Form: $y = a(x-p)(x-q)$ (p and q are x-intercepts)
  - Standard Form: $y = ax^2 + bx + c$
    - x-coordinate of vertex: $-\frac{b}{2a}$
    - y-intercept: c
  - Vertex Form: $y = a(x-h)^2 + k$
    - vertex: (h, k)
  - If a is positive, in any form, this tells you if the curve is upward facing or smiling (so the vertex is a minimum)
  - If a is negative, in any form, this tells you if the curve is downward facing or frowning (so the vertex is a maximum)
  - Any of these equations can be entered into the calculator by using Y=. 2^{nd} TRACE will let you calculate many key attributes including zeros (x-intercepts) and vertices
  - TRACE will give you a y value for every x value

Understand:

- Not every relation is a function.
- A line (created by a linear equation) has a constant slope (positive, negative, zero, or no slope) but a parabola (created by a quadratic equation) has *one change* in slope from positive to negative (downward facing, frown, vertex is a maximum) or negative to positive (upward facing, smile, vertex is a minimum).
- In a linear equation, the highest power is 1. In a quadratic equation, the highest power is 2.
- There are lines that are not functions. Can you think of an example?
- We have seen many examples of functions that have a range that is not all reals. Can you think of an example?

- Slope is vertical change divided by horizontal change.
- Functions have no repeated inputs (x-values) and pass the vertical line test (no vertical line passes through two points).
- Usually interpolation produces a more accurate prediction than extrapolation. The first "test" for how good a prediction may be is to look at R^2. If it is close to 1 (higher than 0.85) and you are interpolating, then the prediction is pretty trustworthy. If extrapolating (with the same R^2), then you need to look at the data and the trend and decide how far out you might be willing to trust.
- The domain is the set of possible x values. The range is the set of possible y values. If we are looking at discrete data (table of values), then these are sets. You will use the STAT button to analyze them by entering them in L1 and L2. If we are looking at continuous data (the best fit lines or curves or any equation that is a function), we will type them into Y1 (using Y= button) to see their behavior.

Be able to do:

- Linear Programming
- Find Key Examples
- Domain, Range, Equations, and Graphs – find connections and fill in any missing pieces
- Solve Linear Systems
- Use function notation
- Factor quadratics to solve
- Regression

Examples to practice:

I. Linear Programming: COOPERATIVE

For each problem, remember to:
a. Define variables.
b. Write a system of inequalities.
c. Graph the system of inequalities in your graphing calculator.
d. Find the coordinates of the vertices of the feasible region.
e. Write an expression to be maximized or minimized.
f. Substitute the coordinates of the vertices in the expression (part e).
g. Select the greatest or least result to answer the problem (depends on problem).

1.) You need to buy some filing cabinets. You know that Cabinet X costs $10 per unit, requires six square feet of floor space, and holds eight cubic feet of files. Cabinet Y costs $20 per unit, requires eight square feet of floor space, and holds twelve cubic feet of files. You have been given $140 for this purchase, though you don't have to spend that much. The office has room for no more than 72 square feet of cabinets. How many of which model should you buy, in order to maximize storage volume?

2.) A calculator company produces a scientific calculator and a graphing calculator. Long-term projections indicate an expected demand of at least 100 scientific and 80 graphing calculators each day. Because of limitations on production capacity, no more than 200 scientific and 170 graphing calculators can be made daily. To satisfy a shipping contract, a total of at least 200 calculators much be shipped each day. If each scientific calculator sold results in a $2 loss, but each graphing calculator produces a $5 profit, how many of each type should be made daily to maximize net profits?

II. Finding Key Examples: INDEPENDENT

Goal: Find key examples for key concepts taught during 1st Semester.

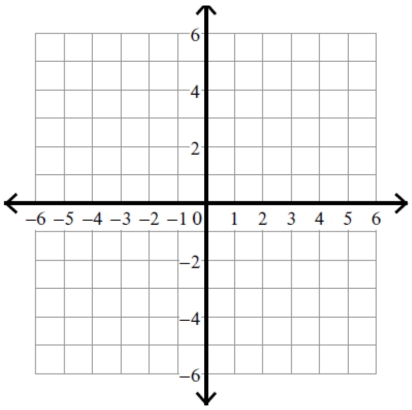
3.) Can you find an example of a function that has a range that is not "all reals"?
   a. You may provide an equation or a diagram. If you choose a diagram, label at least two points.
   b. Explain how you know that the range is not "all reals".

4.) Can you find an example of a line that is a function?
   i. You may provide an equation or a diagram. If you choose a diagram, label at least two points.
   ii. Explain how you know that the line is a function.

III. Fill in the missing part: INDEPENDENT

It could be domain, range, or equation. Fill in whatever "blanks" are here!

5.)

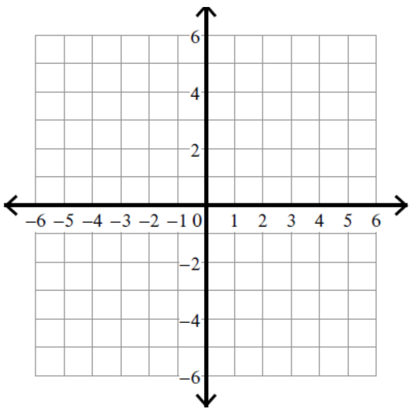


DOMAIN: ________________

RANGE: __________________

EQUATION: $5x - 2y = 10$

6.)

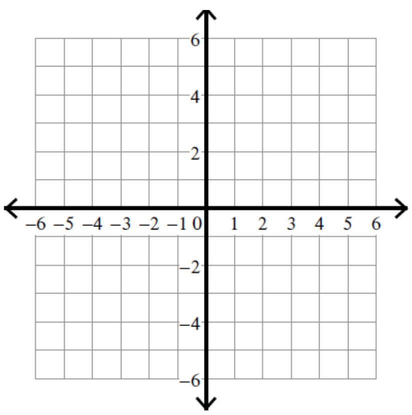


DOMAIN: ________________

RANGE: __________________

EQUATION: $y = -4$

7.)

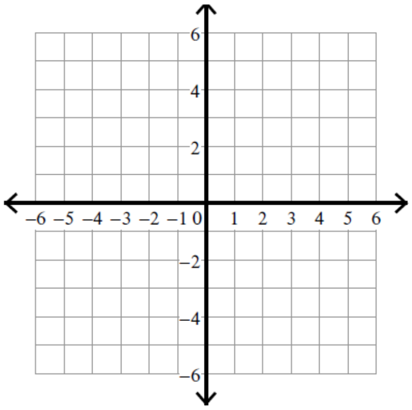


DOMAIN: ________________

RANGE: __________________

EQUATION: $x = 5$

8.)



DOMAIN: ________________

RANGE: _________________

EQUATION: $x-2y=-4$

IV. Solve Systems of Linear Equations: INDEPENDENT

Show me your matrix skills to find the variable values for each linear system! Remember to rearrange so all variables and their coefficients are on the left, lined up and so all the numbers are on the right. Remember that if a variable is not there, you need a 0 in its place to have the same meaning. Examples of nicely lined up systems are numbers 9 and 12. The coefficient matrix is A (type MATRX then Edit and the numbers, called constant matrix, are matrix B. Then, after they are entered, you can find the variable values by typing MATRX again then A^{-1} * *B. The "-1" button is marked* x^{-1}.

9.) x + 7y = 0
2x – 8y = 22

10.) 9 – 4x = -y
3 = x – y

11.) x + 7y = 0
2x = 22 + 8y

12.) x + 4y – 4z = 4
-5x + 0y + z = -25
-2x + 0y – 5z = 17

13.)
$$-6x+6y=6$$
$$-6x+3y=-12$$

14.)
$$-x-7y=14$$
$$-4x-14y=28$$

15.)
$$4x + 4y + z = 24$$
$$2x - 4y + z = 0$$
$$5x - 4y - 5z = 12$$

16.)
$$x = -4y + 4z + 4$$
$$z = 5x - 25$$
$$-2x - 5z = 17$$

17.)
$$z = -4x + 4y + 13$$
$$x + 2y - 2z = 10$$
$$x = 2z + 10$$

V. Use Function Notation: INDEPENDENT

Substitute x-values into expressions to solve for f(x).

18.) $f(x) = -2x^2 - 1$. Show organized work to find f(-3)

19.) $f(x) = -3x^2 + 6$. Show organized work to find f(-1)

20.) $f(x) = 2x^2 - 1$. Show organized work to find f(2)

21.) $f(x) = -|x| - 2$. Show organized work to find f(-2)

22.) $f(x) = -3|x| + 5$. Show organized work to find f(1)

23.) $f(x) = 3|x| + 5$. Show organized work to find f(-1)

VI. Factor to Solve: INDEPENDENT

Factor quadratics to solve them. First, be sure to set the quadratic equal to 0!

24.) $(3n - 2)(4n + 1) = 0$

25.) $x^2 = -18 - 9x$

VII. Regression: INDEPENDENT

Use Stat Edit to enter data. Then use Stat Calc to calculate regression (remember to 2^{nd} Catalog, turn DiagnoticOn). Get R2, fit line or curve to data, evaluate goodness of fit, predict and interpret predictions, find key points on curve or line, and decide based on R2 and interpolation vs. extrapolation whether or not a line or a quadratic is a good fit for the data. If the data is linear, be prepared to describe its correlation as strong/positive, weak/positive, strong/negative, weak/negative. If the data is quadratic, be ready to interpret the meaning of its vertex in context

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26.) From Quiz 2c Review:

Table: Distance that Hare is from start during the turtle vs. the hare race
Source: Aesop's Fables

Time (seconds)	**Distance from Start (feet)**
0	0
5	6
15	25
30	45
40	30
45	10

a.) Looking at the data set alone – before plotting – why does this data seem to be quadratic? ____

b.) What is the independent variable? ___________________________

c.) What is the dependent variable? ___________________________

d.) Is this data set discrete or continuous? ___________________________

e.) What is the domain of this data set? _______________________________________

f.) What is the range of this data set? _______________________________________

g.) Now, input this data and find the quadratic regression curve. Then, answer the following questions.

   a. Please fill in the following blanks for the quadratic equation.
      a = ______________ b = ______________ c = ______________
   b. What is the equation for the quadratic equation that best fits this data set?
      $f(x)$ = ____
   c. Is this curve discrete or continuous? ___________________________

h.) Plot this curve with the data on the same graph. Set up the window as needed so that you can see both the x and y axes. Then, answer the following questions.

   a. Find the two x-intercepts (or two zeros):
   b. Find the vertex. ____
   c. What is the domain of the quadratic regression curve?
   d. What is the range of the quadratic regression curve?
   e. What is the y-intercept?
   f. At what time did the hare reach his farthest point from the start? Round to the nearest tenth of a second.
   g. Use your model to predict the hare's distance from the start after 35 seconds. Show organized work.

.

27.)

Table: Distance that Turtle is from start during the turtle vs. the hare race
Source: Aesop's Fables

Time (seconds)	**Distance from Start (feet)**
0	0
5	6
15	15
30	25
40	30
45	36

a.) Looking at the data set alone – does this data seem quadratic or no?
b.) What is the independent variable? ___________________________
c.) What is the dependent variable? ___________________________
d.) Is this data set discrete or continuous? ___________________________
e.) What is the domain of this data set? _______________________________________
f.) What is the range of this data set? _______________________________________
g.) Now, input this data and find the best fit for this data set. Then, answer the following questions. USE Y2 to place continuous curve or you will override your work from 26!!!!!!!!!!!!!
   i.) $f(x)$ = ____
   ii.) Is this curve discrete or continuous? ___________________________
h.) Plot this curve with the data on the same graph. Set up the window as needed so that you can see both the x and y axes. Then, answer the following questions.
   i.) Find the x-intercept
   ii.) What is the domain of f(x)?
   iii.) What is the range of f(x)?
   iv.) What is the y-intercept? State units.
   v.) What is the slope? State units.
   vi.) If the race was two minutes, 10.8 seconds long, how many feet did the turtle run? Round to the nearest tenth of a foot.
   vii.) Use your model to predict the turtle's distance from the start after 35 seconds.
   viii.) Use your model to predict the turtle's distance from the start after 0.5 minutes.

28.) Now combine! Consider 2^{nd} Calc Intersection!!!

Who wins?

When do they cross, if they do?

After 15 seconds, who would you think would win?

What about after 25 seconds?

What about after 40 seconds?

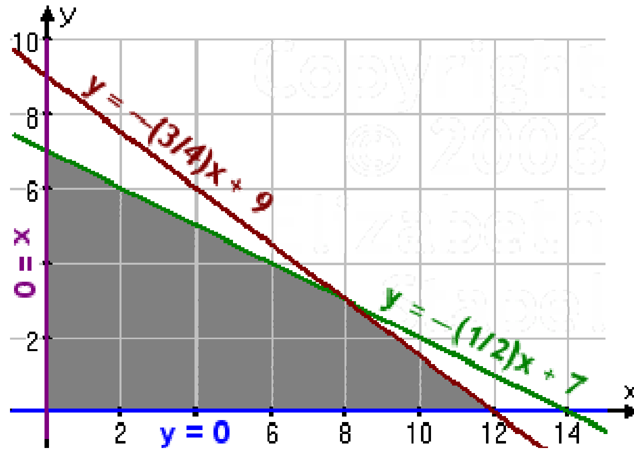
Solutions to Problems to Practice:

1.) The question asks for the number of cabinets I need to buy, so my variables will stand for that:

x: number of model X cabinets purchased

y: number of model Y cabinets purchased

Naturally, $x \geq 0$ and $y \geq 0$. I have to consider costs and floor space (the "footprint" of each unit), while maximizing the storage volume, so costs and floor space will be my constraints, while volume will be my optimization equation.



cost: $10x + 20y \leq 140$, or $y \leq -(\,^{1}/_{2}\,)x + 7$
space: $6x + 8y \leq 72$, or $y \leq -(\,^{3}/_{4}\,)x + 9$
volume: $V = 8x + 12y$

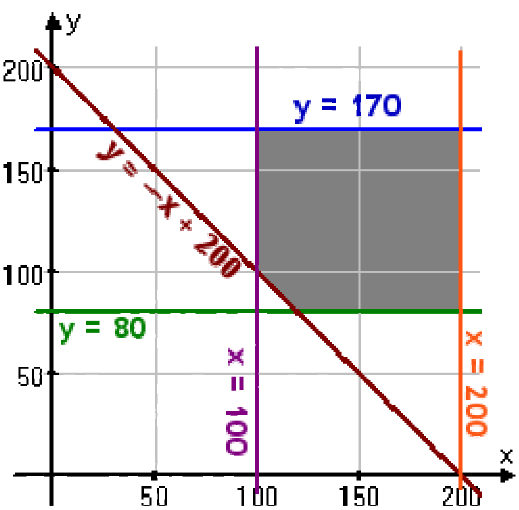
This system (along with the first two constraints) graph is shown to the right:

When you test the corner points at (8, 3), (0, 7), and (12, 0), you should obtain a maximal volume of 100 cubic feet by buying eight of model X and three of model Y.

2.) The question asks for the optimal number of calculators, so my variables will stand for that:

x: number of scientific calculators produced

y: number of graphing calculators produced



Since they can't produce negative numbers of calculators, I have the two constraints, $x \geq 0$ and $y \geq 0$. But in this case, I can ignore these constraints, because I already have that $x \geq 100$ and $y \geq 80$. The exercise also gives maximums: $x \leq 200$ and $y \leq 170$. The minimum shipping requirement gives me $x + y \geq 200$; in other words, $y \geq -x + 200$. The revenue relation will be my optimization equation: $R = -2x + 5y$. So the entire system is:

$R = -2x + 5y$, subject to:
$100 \leq x \leq 200$
$80 \leq y \leq 170$
$y \geq -x + 200$

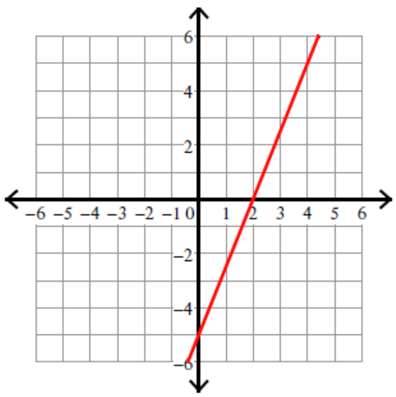
The feasibility region graph is to the right.

When you test the corner points at (100, 170), (200, 170), (200, 80), (120, 80), and (100, 100), you should obtain the maximum value of $R = 650$ at $(x, y) = (100, 170)$. That is, the solution is "100 scientific calculators and 170 graphing calculators".

3.) An example of a function that has a range that is not all reals was studied for most of quarter 2; a quadratic. a.) $y = x^2$ graphs as an upward facing parabola with a minimum (vertex) at the origin (0, 0). b.) The range of $y = x^2$ is $y \geq 0$.

4.) Most lines are functions. Lines with a zero slope (horizontal lines, m = 0), positive slope (m is positive), or negative slope (m is negative) are all functions because they pass the vertical line test. As long as I do not choose the no slope case (like x = 5, which is a vertical line), I am good. a.) So I will choose $y = 3x - 5$. This starts at (0, -5) and has another point up 3, right 1 from this point, so (1, -2). b.) This is a function because it passes the vertical line test.

5.)

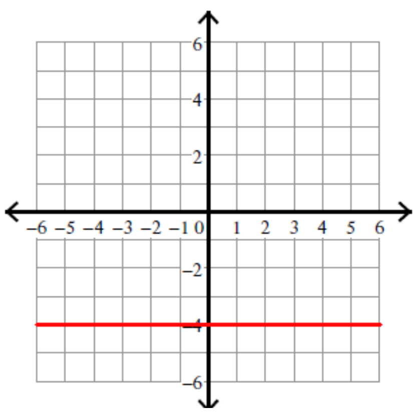


DOMAIN: <u>all reals</u>

RANGE: <u>all reals</u>

EQUATION: $5x - 2y = 10$

6.)

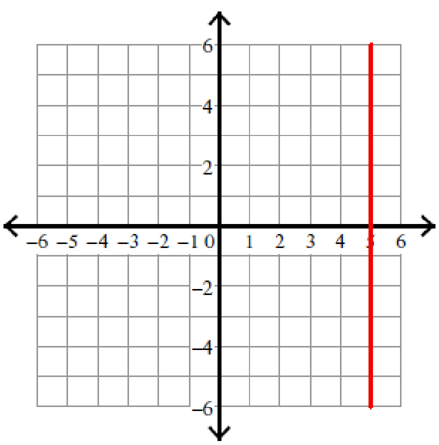


DOMAIN: <u>all reals</u>

RANGE: <u>y = -4</u>

EQUATION: $y = -4$

7.)

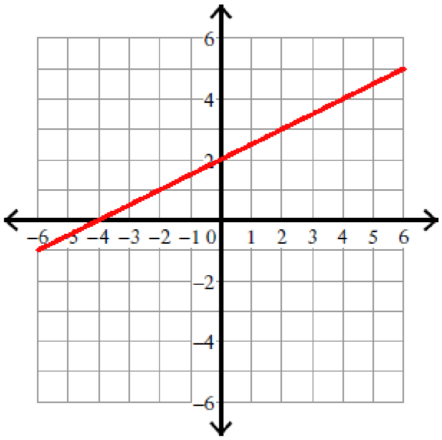


DOMAIN: <u>x = 5</u>

RANGE: <u>all reals</u>

EQUATION: $x = 5$

8.)



DOMAIN: <u>all reals</u>

RANGE: <u>all reals</u>

EQUATION: $x - 2y = -4$

9.) (6, -7)   10.) (2, -1)   11.) (7, -1)   12.) (4, 5, -5)

13.) Matrix of coefficients: $\begin{bmatrix} -6 & 6 \\ -6 & 3 \end{bmatrix}$ solution: x = 5, y = 6

Matrix of Constants: $\begin{bmatrix} 6 \\ -12 \end{bmatrix}$

14.) Matrix of coefficients: $\begin{bmatrix} -1 & -7 \\ -4 & -14 \end{bmatrix}$ solution: x = 0, y = 2

Matrix of Constants: $\begin{bmatrix} 14 \\ 28 \end{bmatrix}$

15.) Matrix of coefficients: $\begin{bmatrix} 4 & 4 & 1 \\ 2 & -4 & 1 \\ 5 & -4 & -5 \end{bmatrix}$ solution: x = 4, y = 2, z = 0

Matrix of Constants: $\begin{bmatrix} 24 \\ 0 \\ 12 \end{bmatrix}$

16.) (4, -5, 5)   17.) (4, 0, -3)   18.) -19   19.) 3   20.) 7

21.) -4   22.) 2   23.) 8   24.) n = {2/3, -1/4}   25.) {-6, -3}

26.)
a.) It goes up and then down again. b.) Time (sec) c.) Distance from start (ft)
d.) DISCRETE e.) {0, 5, 15, 30, 40, 45} f.) {0, 6, 10, 25, 30, 45}
g.) a: a = -0.067, b = 3.46. c = -5.42 b.: $f(x) = -0.067x^2 + 3.46x - 5.42$
c.: CONTINUOUS Used QuadReg L1, L2, Y1 for g!
h.) a: 1.62 and 50.24 b.: (25.93, 39.44) c.: all reals d.: $y \le 39.44$
e.: $f(0) = -0.067(0)^2 + 3.46(0) - 5.42 = -5.42$ feet
(the hare moved more backward than forward overall!)
g.:The farthest point from the start happens at the maximum, where x = 25.93 sec
h.: $f(35) = -0.067(35)^2 + 3.46(35) - 5.42$ or trace and type x = 35 to get y ≈ 33.95 ft
.
27.)

a.) It does not. The distance from start is increasing steadily as time increases.
b.) Time (sec) c.) Distance from start (ft) d.) discrete
e.) {0, 5, 15, 30, 40, 45} f.) {0, 6, 15, 25, 30, 36}
g.) i.) LinReg L1, L2, Y1 gives a ≈ .75, b ≈ 1.77, with r^2 ≈.99 ii.) contin.
h.) i.) ≈-2.35 (used find zero functionality) ii.) all reals iii.) all reals
iv.) ≈ 1.77 feet v.) ≈ .75 feet per second
vi) two minutes is 120 seconds. TRACE to 130.8 seconds produces ≈ 100.0 feet from the start. vii.) 28.1 feet viii.) 0.5 minutes = 30 sec; 24.3 feet

28.) Now combine!

Who wins? The race is 100 feet long. The turtle is the only animal that reaches 100 feet because the hare is a downward facing parabola that turns around before it can reach. Its vertex y value is less than 100.

When do they cross, if they do? At 5.48 seconds and again at 39.15 seconds. 2^{nd} Calc Intersection helps!

After 15 seconds, who would you think would win? The hare was in the lead!

What about after 25 seconds? The hare was still in the lead!

What about after 40 seconds? The turtle took the lead .85 seconds earlier!